Assess the risk of pit stopping, using probability and Game Theory with Sequential Game Decision Making.

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In game theory, a (game) tree is a graph whose nodes are positions in a game and whose edges are moves. The complete tree for a game is the tree starting at the initial position and containing all possible moves from each position. Game tree models allow players to make better decisions by forcing them to consider the actions and reactions of all other players involved. In a sequential game, the decision-maker eliminates a great deal of uncertainty simply by creating a clear-cut list of the various players, their actions and reactions, and the decision-maker's best response to each. This method is usefull in a wide range of applications, from business to sport.

Here is a very simple application in car racing to explain how different opportunities related with probability could be usefull for better decisions in case of pit-stops.

Defining *n* as the number of drivers. They need to choose the action A∊{Pit, Not pit}. The driver position is between 1 and *n*, ∊{1,*n*}. A general *kth* position is located in this sequence: 1 , … , *k*-1 , *k* , *k*+1 , … , *n*. Defining *j* as another driver on the track, racing near *k* (for more details, read the following ***b)*** assumption). Let driver 1 *k* in first position and driver 2 *j* in second position. We assume that this two drivers are isolated, so the driver in 3rd position is very far away (more than time needed for a regular pit-stop). If both driver pit, pits are in a very close time.

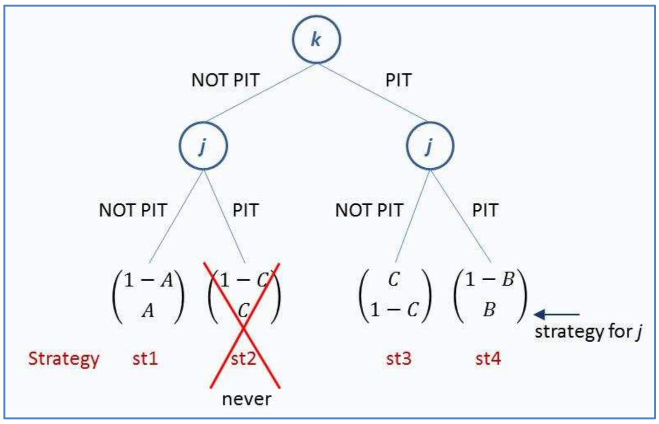
Let the following assumptions: ***a)*** if and *j* choose different strategies, the driver who do not pit will be ahead the driver who pit; ***b)*** if and *j* choose the same strategy, *k* will be ahead *j* if *k*<*j* (i.e. before the pit-stop *k* is ahead *j*).

At this point at the restarting we have three possible situations with different probability of occurrence:

- A, it is the probability a car that did not pit successfully passes another car that did not pit;
- B, it is the probability a car that did pit successfully passes another car that did pit;
- C, it is the probability a car that did pit successfully passes another car that did not pit;
- D, it is the probability a car that did not pit successfully passes another car that did pit; this occurrence has probability=1 because of point ***a)***. So this is not an option.

CASE	DRIVER		PROBABILITY OF PASSING
	1	2	
A	NO PIT	NO PIT	
B	PIT	PIT	
C	PIT	NO PIT	
D	NO PIT	PIT	independent by pits

The game tree is the following:

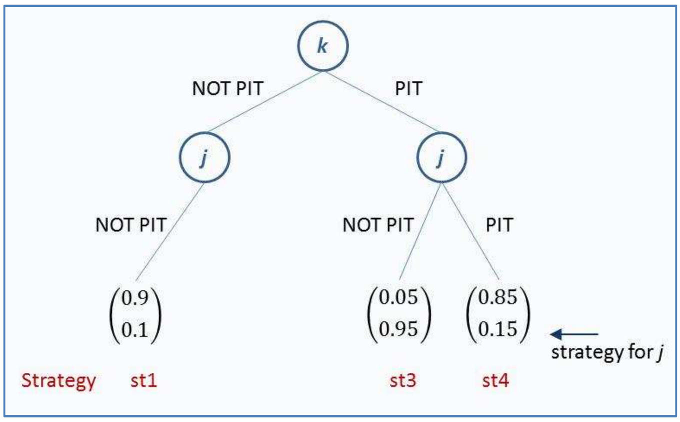


Because of ***a)*** and ***b)***, driver 1 in strategies st1, st2 and st4 will be ahead and the chances to win are 1 minus the probability that he is passed. In strategy st3 driver 1 must pass driver 2. The order of decisions is essential. And is essential to define the right probabilities (success of pass). In this case we have many parameters available: tyres type, tyres degradation, aerodynamic, fuel consumption, and others. During the races probability changes quickly (some variables are very random and often correlated with other variables; for example tyres degradation with aerodynamic) and this is the reason why frequently strategies fail.

Exemplification.

CASE	strategy			positions after strategy	gap after strategy	PROBABILITY OF PASSING	Notes
		k	j				
A	st1	NO PIT	NO PIT	k --->j	2'	10%	
B	st4	PIT	PIT	k --->j	2'	15%	because of the uncertainty of pits
C	st3	PIT	NO PIT	j --->k	18'	5%	with pit, tyres are new with more performance

The tree will be



If driver *k* will pit, driver *j* must not pit. This could be obvious, even if probability must be carefully calculated due to the high number of variables, and because of the instability of this variables. But what is important to notice is the little difference between st1 and st4. These are the only two situations in which *k* and *j* do the same action. If *k* not pit and *j* needs a pit, no matter with strategies. Related with the choice of *k* (pit or not pit) the real chance of *j* is to redefine variables (their values) in order to get different probability. To be precise, if *k* pit, and *j* needs a pit too (i.e. *j* can't hope in winning st3 strategy) probability in st4 must be lower than st1.

Due to the rules and the cars in Formula E, a particular game tree is needed because variables are less and different.

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